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MATHEMATICS **9709/13**

Paper 1 **May/June 2017**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	$7C1 \times 2^6 \times a(x)$, $7C2 \times 2^5 \times [a(x)]^2$	B1 B1	SOI Can be part of expansion. Condone ax^2 only if followed by a^2. ALT $2^7[1+ax/2]^7 \to 7C1[a(x)/2] = 7C2[a(x)/2]^2$
	$a = \frac{7\times 2^6}{21\times 2^5} = \frac{2}{3}$	B1	Ignore extra soln $a = 0$. Allow $a = 0.667$. Do not allow an extra x in the answer
	Total:	**3**	

Question	Answer	Marks	Guidance
2(i)	$S = \frac{r^2 - 3r + 2}{1-r}$	M1	
	$S = \frac{(r-1)(r-2)}{1-r} = \frac{-(1-r)(r-2)}{1-r} = 2 - r$ OR $\frac{(1-r)(2-r)}{1-r} = 2 - r$ OE	A1	**AG** Factors must be shown. Expressions requiring minus sign taken out must be shown
	Total:	**2**	
2(ii)	Single range $1 < S < 3$ or (1, 3)	B2	Accept $1 < 2 - r < 3$. Correct range but with $S = 2$ omitted scores SR **B1** $1 \leqslant S \leqslant 3$ scores SR **B1**. [$S > 1$ and $S < 3$] scores SR **B1**.
	Total:	**2**	

Question	Answer	Marks	Guidance
3	**EITHER** Elim *y* to form 3-term quad eqn in $x^{1/3}$ (or *u* or *y* or even *x*)	**(M1**	Expect $x^{2/3}-x^{1/3}-2(=0)$ or $u^2-u-2\ (=0)$ etc.
	$x^{1/3}$ (or *u* or *y* or *x*) = 2, –1	***A1**	Both required. But $\underline{\boldsymbol{x}}$ = 2,–1 and not then cubed or cube rooted scores **A0**
	Cube solution(s)	**DM1**	Expect *x* = 8, – 1. Both required
	(8, 3), (–1,0)	**A1)**	
	OR Elim *x* to form quadratic equation in *y*	**(M1**	Expect $y+1=(y-1)^2$
	$y^2-3y=0$	***A1**	
	Attempt solution	**DM1**	Expect *y* = 3, 0
	(8, 3), (–1,0)	**A1)**	
	Total:	**4**	

Question	Answer	Marks	Guidance
4(i)	$\overrightarrow{OB}-\overrightarrow{OA}\left(=\overrightarrow{AB}\right)=\begin{pmatrix}5\\4\\-3\end{pmatrix}-\begin{pmatrix}5\\1\\3\end{pmatrix}=\begin{pmatrix}0\\3\\-6\end{pmatrix}$	**B1**	
	$\overrightarrow{OP}=\begin{pmatrix}5\\1\\3\end{pmatrix}+\frac{1}{3}\begin{pmatrix}0\\3\\-6\end{pmatrix}=\begin{pmatrix}5\\2\\1\end{pmatrix}$	**M1 A1**	If $\overrightarrow{OP}$ not scored in (i) can score SR **B1** if seen correct in (ii). Other equivalent methods possible
	Total:	**3**	
4(ii)	Distance $OP=\sqrt{5^2+2^2+1^2}\ =\ \sqrt{30}$ or 5.48	**B1 FT**	FT on *their* $\overrightarrow{OP}$ from (i)
	Total:	**1**	
4(iii)	Attempt $\overrightarrow{AB}.\overrightarrow{OP}$. Can score as part of $\overrightarrow{AB}.\overrightarrow{OP}=(AB)(OP)\cos\theta$ Rare ALT: Pythagoras $\lvert\overrightarrow{OP}\rvert^2+\lvert\overrightarrow{AP}\rvert^2=5+30=\lvert\overrightarrow{OA}\rvert^2$	**M1**	Allow any combination of $\overrightarrow{AB}.\overrightarrow{PO}$ etc. and also if $\overrightarrow{AP}$ or $\overrightarrow{PB}$ used instead of $\overrightarrow{AB}$ giving 2–2 = 0 & 4–4 = 0 respectively. Allow notation × instead of **.** .
	(0 + 6 – 6) = 0 hence perpendicular. (Accept 90º)	**A1 FT**	If result not zero then 'Not perpendicular' can score **A1**FT if value is 'correct' for *their* values of $\overrightarrow{AB},\overrightarrow{OP}$ etc. from (i).
	Total:	**2**	

Question	Answer	Marks	Guidance
5(i)	$\dfrac{2\sin\theta+\cos\theta}{\sin\theta+\cos\theta}=\dfrac{2\sin\theta}{\cos\theta}$	**M1**	Replace $\tan\theta$ by $\sin\theta/\cos\theta$
	$2\sin\theta\cos\theta+\cos^2\theta=2\sin^2\theta+2\sin\theta\cos\theta \Rightarrow c^2=2s^2$	**M1 A1**	Mult by c(s + c) or making this a common denom.. For **A1** simplification to **AG** without error or omission must be seen.
	Total:	**3**	
5(ii)	$\tan^2\theta=1/2$ or $\cos^2\theta=2/3$ or $\sin^2\theta=1/3$	**B1**	Use $\tan\theta=s/c$ or $c^2+s^2=1$ and simplify to one of these results
	$\theta=35.3°$ or $144.7°$	**B1 B1 FT**	FT for 180 – other solution. SR **B1** for radians 0.615, 2.53 (0.196π, 0.804π) Extra solutions in range amongst solutions of which 2 are correct gets **B1B0**
	Total:	**3**	

Question	Answer	Marks	Guidance
6	Gradient of normal is $-1/3 \to$ gradient of tangent is 3 SOI	**B1 B1 FT**	FT from *their* gradient of normal.
	$dy/dx = 2x - 5 = 3$	**M1**	Differentiate and set = *their* 3 (numerical).
	$x = 4$	***A1**	
	Sub $x = 4$ into line $\to y = 7$ & sub *their* (4, 7) into curve	**DM1**	OR sub $x = 4$ into curve $\to y = k - 4$ and sub *their*$(4, k - 4)$ into line OR other valid methods deriving a linear equation in k (e.g. equating curve with either normal or tangent and sub $x = 4$).
	$k = 11$	**A1**	
	Total:	**6**	

Question	Answer	Marks	Guidance
7(i)	$\sin ABC = 8/10 \rightarrow ABC = 0.927(3)$	B1	Or cos = 6/10 or tan = 8/6. Accept 0.295π.
	Total:	**1**	
7(ii)	$AB = 6(\text{Pythagoras}) \rightarrow \Delta BCD = 8 \times 6 = 48.0$	M1A1	**OR** 8×10sin0.6435 or ½×10×10sin((2)×0.927)=48. 24or 40or80 gets **M1A0**
	Area sector $BCD = ½ \times 10^2 \times (2) \times \textit{their}\, 0.9273$	*M1	Expect 92.7(3). 46.4 gets **M1**
	Area segment = 92.7(3) – 48	*A1	Expect 44.7(3). Might not appear until final calculation.
	Area semi-circle – segment = $½ \times \pi \times 8^2 - \textit{their}(92.7 - 48)$	DM1	Dep. on previous **M1A1** OR $\pi \times 8^2 - (½ \times \pi \times 8^2 + \textit{their}\, 44.7)$.
	Shaded area = 55.8 – 56.0	A1	
	Total:	**6**	

Question	Answer	Marks	Guidance
8(i)	$(b-1)/(a+1)=2$	**M1**	OR Equation of *AP* is $y-1=2(x+1)\to y=2x+3$
	$b=2a+3$ CAO	**A1**	Sub $x=a$, $y=b$ $\to b=2a+3$
	Total:	**2**	
8(ii)	$AB^2=11^2+2^2=125$ oe	**B1**	Accept $AB=\sqrt{125}$
	$(a+1)^2+(b-1)^2=125$	**B1 FT**	FT on *their* 125.
	$(a+1)^2+(2a+2)^2=125$	**M1**	Sub from part (i) → quadratic eqn in *a* (or possibly in *b* → $b^2-2b-99=0$)
	$(5)(a^2+2a-24)=0\to$ eg$(a-4)(a+6)=0$	**M1**	Simplify and attempt to solve
	$a=4$ or -6	**A1**	
	$b=11$ or -9	**A1**	OR (4, 11), (–6, –9) If **A0A0**, SR1 for either (4, 11) or (–6, –9)
	Total:	**6**	

Question	Answer	Marks	Guidance
9(i)	$(3x-1)^2+5$	**B1B1B1**	First 2 marks dependent on correct $(ax+b)^2$ form. OR $a=3,\ b=-1,\ c=5$ e.g. from equating coefs
	Total:	**3**	
9(ii)	Smallest value of p is 1/3 seen. (Independent of (i))	**B1**	Allow $p \geqslant 1/3$ or $p=1/3$ or 1/3 seen. But not in terms of x.
	Total:	**1**	
9(iii)	$y=(3x-1)^2+5 \Rightarrow 3x-1=(\pm)\sqrt{y-5}$	**B1 FT**	OR $y=9\left(x-\frac{1}{3}\right)^2+5 \Rightarrow (y-5)/9=\left(x-\frac{1}{3}\right)^2$ (Fresh start)
	$x=(\pm)\frac{1}{3}\sqrt{y-5}+\frac{1}{3}$ OE	**B1 FT**	Both starts require 2 operations for each mark. FT for *their* values from part (i)
	$\mathrm{f}^{-1}(x)=\frac{1}{3}\sqrt{x-5}+\frac{1}{3}$ OE domain is $x \geq$ *their*5	**B1B1 FT**	Must be a function of x and $\pm$ removed. Domain must be in terms of x. Note: $\sqrt{y-5}$ expressed as $\sqrt{y}-\sqrt{5}$ scores Max **B0B0B0B1** [See below for general instructions for different starts]
	Total:	**4**	
9(iv)	$q<5$ CAO	**B1**	
	Total:	**1**	

Alt 9(iii)
For start $(ax-b)^2+c$ or $a(x-b)^2+c$ ($a \neq 0$) ft for their a, b, c
For start $(x-b)^2+c$ ft but award only **B1** for 3 correct operations
For start $a(bx-c)^2+d$ ft but award **B1** for first2 operations correct and **B1** for the next 3 operations correct

Question	Answer	Marks	Guidance
10(a)(i)	Attempt to integrate $V = (\pi)\int (y+1)\,\mathrm{d}y$	M1	Use of h in integral e.g. $\int (h+1) = ½h^2 + h$ is **M0**. Use of $\int y^2 \mathrm{d}x$ is **M0**
	$= (\pi)\left[\dfrac{y^2}{2} + y\right]$	A1	
	$= \pi\left[\dfrac{h^2}{2} + h\right]$	A1	**AG**. Must be from clear use of limits 0→ h somewhere.
	Total:	3	
10(ii)	$\int (y+1)^{1/2}\,\mathrm{d}y$ **ALT** $6 - \int (x^2 - 1)\,\mathrm{d}x$	M1	Correct variable and attempt to integrate
	$⅔(y+1)^{3/2}$ oe **ALT** $6 - (⅓x^3 - x)$ CAO	*A1	Result of integration must be shown
	$⅔[8-1]$ **ALT** $6 - [\left(\dfrac{8}{3} - 1\right) - \left(\dfrac{1}{3} - 1\right)]$	DM1	Calculation seen with limits 0→3 for y. For ALT, limits are 1→2 and rectangle.
	14/3 **ALT** 6 – 4/3 = 14/3	A1	16/3 from $⅔ \times 8$ gets **DM1A0** provided work is correct up to applying limits.
	Total:	4	

Question	Answer	Marks	Guidance
10(b)	Clear attempt to differentiate wrt h	**M1**	Expect $\frac{\mathrm{d}V}{\mathrm{d}h} = \pi(h+1)$. Allow $h + 1$. Allow h.
	Derivative = 4π SOI	***A1**	
	$\frac{2}{their\ \text{derivative}}$. Can be in terms of h	**DM1**	
	$\frac{2}{4\pi}$ or $\frac{1}{2\pi}$ or 0.159	**A1**	
	Total:	**4**	

Question	Answer	Marks	Guidance
11(i)	$\text{f}'(x)=[(4x+1)^{1/2}\div ½]\ [\div 4]\ (+c)$	**B1 B1**	Expect $½(4x+1)^{1/2}\ (+c)$
	$\text{f}'(2)=0 \Rightarrow \frac{3}{2}+c=0 \Rightarrow c=-\frac{3}{2}$ (Sufficient)	**B1 FT**	Expect $½(4x+1)^{1/2}-\frac{3}{2}$. FT on *their* $\text{f}'(x)=k(4x+1)^{1/2}+c$. (i.e. $c=-3k$)
	Total:	**3**	
11(ii)	$\text{f}''(0)=1$ SOI	**B1**	
	$\text{f}'(0)=1/2-1½=-1$ SOI	**B1 FT**	Substitute $x = 0$ into *their* f $'(x)$ but must not involve c otherwise **B0B0**
	$\text{f}(0)=-3$	**B1 FT**	FT for 3 terms in AP. FT for 3rd **B1** dep on 1st **B1**. Award marks for the AP method only.
	Total:	**3**	
11(iii)	$\text{f}(x)=\left[½(4x+1)^{3/2}\div 3/2\div 4\right]-\left[1½x\right]\ (+k)$	**B1 FT** **B1 FT**	Expect $(1/12)(4x+1)^{3/2}-1½x\ \ (+k)$. FT from *their* f $'(x)$ but c numerical.
	$-3=1/12-0+k \Rightarrow k=-37/12$ CAO	**M1A1**	Sub $x=0, y=$ *their* $\text{f}(0)$ into *their* f(x). Dep on cx & k present (c numerical)
	Minimum value = f(2) = $\frac{27}{12}-3-\frac{37}{12}=-\frac{23}{6}$ or -3.83	**A1**	
	Total:	**5**	